

The results of these investigations are given in tables 1 and 2 in the form of pressure-density isotherms, and are shown graphically in fig. 2 and 3.

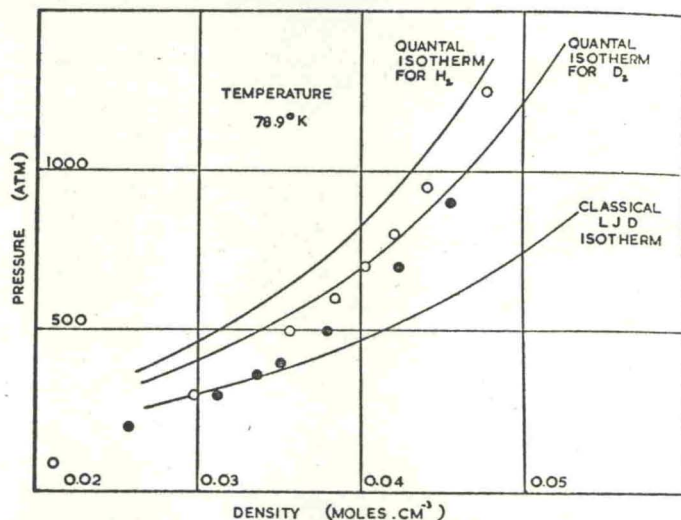


FIG. 3.—Theoretical and experimental isotherms at 78.9° K. The open circles are experimental points for H₂, the filled circles are for D₂.

DISCUSSION

In the figures, our results are compared with the classical LJD and quantal isotherms for H₂ and D₂ at 64.5° K and 78.9° K. The theoretical curves have been calculated assuming that the potential energy between a pair of molecules of either H₂ or D₂ is represented by eqn. (1) with $\epsilon^* = 50.75 \times 10^{-16}$ ergs, and $D = 2.92 \times 10^{-8}$ cm. These parameters were derived by de Boer and Michels from a quantal analysis of the second virial coefficients of H₂ and D₂. The assumption of the identity of the fields between H₂ and D₂ molecules is justified by the similarity in behaviour of these gases⁷ when the temperature is high enough to render the quantum effects unimportant. We have interpolated in the table of Wentorf, Buehler, Hirschfelder and Curtiss³ to find the classical LJD pressures and have added to these the contributions of eqn. (2) to obtain the quantal isotherms.

It is apparent from fig. 2 and 3 that the experimental points lie closer to the quantal curves than to the classical isotherms, particularly at the higher densities where the LJD model should apply more accurately. This fact alone, however, is not a convincing proof of the reality of the quantal inflation of the pressure because the classical LJD theory might be inaccurate at these reduced pressures and reduced temperatures. A more significant fact, therefore, is that the difference in the pressures of H₂ and D₂ at a given density is roughly that predicted by the quantal theory. If the gases behaved classically there would, of course, be no difference.

The difficulty mentioned in the previous paragraph can be met by plotting the data for H₂ and D₂ on a reduced scale together with the results for other simple non-polar gases and the reduced LJD isotherms. It is then possible to compare directly the behaviour of the heavy and light gases and the predictions of the classical and quantal theories. To do this, we introduce the quantities:

$$\begin{aligned} \text{reduced pressure, } p^* &= pv_0/N\epsilon^* = pD^3/\epsilon^*, \\ \text{reduced volume, } v^* &= v/v_0 = v/ND^3, \\ \text{reduced temperature, } T^* &= kT/\epsilon^*, \end{aligned}$$

defined by Wentorf, Buehler, Hirschfelder and Curtiss.³ It is also convenient to write the quantal correction (2) in terms of the dimensionless parameters

$\epsilon^* = h^2 D(m\epsilon^*)^{-1/2}$ which is the reduced equation

$$\Delta p^* = T^* \left[v^* \right]$$

where y^* is a function of

$$(1 + 12y^* + 25 \cdot 2y^{*2})$$

in conformity with the quantal equation of state

where f is a universal function. For the general plot temperature $T^* = 2.14 \epsilon^*$

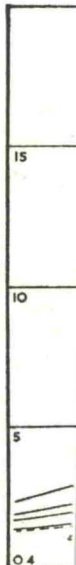


FIG. 4.—Reduced isotherms.

He
H₂
D₂
N₂
A

H₂ and D₂. The data have been taken from the experimental data for the temperature scale of Betherington, Kvalne and A.